**Q4：**

(1) Keep rolling

P(payoff=0)=1/6

5/6 chance, one can gain 4(Expected) more, since 35 ==> 37,38,39,40,41, one can continue to roll a second time, and won't hit 49, so the Expected value>(35+4+3.5)\*5/6>35.

(2) Rolling a dice infinitely and calculated the accumulated number, P(6)>P(5)>P(4)>P(3)>P(2)>P(1)=1/6, and

p(1)=1/6

p(2)=1/6+1/36=7/36

p(3)=1/216+2/36+1/6=49/216

[111, 12, 21, 3]

p(4)=1/1296+3/216+3/36+1/6=343/1296

[1111,112,121,211,22,13,31,4]

p(5)=1/7776+4/1296+6/216+4/36+1/6= 2401/7776

[11111,1112,1121,1211,2111,113,131,311,14,41,122,212,221,23,32,5]

p(6)=1/46656+5/7776+10/1296+10/216+5/36+1/6=16807/46656

[111111,11112(5),1113(4),114(3),15(2),6,2211(6),222,123(6),24(2),33]

P(7)=[P(6)+P(5)+P(4)+P(3)+P(2)+P(1)]/6=0.253

P(8)=[P(6)+P(5)+P(4)+P(3)+P(2)+P(7)]/6=p(1)/36+[P(6)+P(5)+P(4)+P(3)+P(2)]\*7/36=0.268

p(9)=[P(6)+P(5)+P(4)+P(3)+P(8)+P(7)]/6=P(8)-P(2)/6+P(8)/6=0.280

p(10)=0.289

p(11)=0.293

p(12)=0.290

p(13)=0.279

starting from 37/38/39/40/41

stop at 44,45,46,47,48,49(0)

43:44-49 equal posiblity

43:[p(6)+p(5)+p(4)+p(3)+p(2)]/6

42:1/6=>43=>44-49(equal)

1/6=>44-48

42:[p(1)+p(5)+p(4)+p(3)+p(2)]/6

41:p(2)=>43=44-49(equal)

p(1)=>42

1/6=>44-47

41:[p(1)+p(4)+p(3)+p(2)+1]/6

40:p(3)=>43=>44-49(equal)

p(2)=>42

p(1)=>41

1/6 =>44-46

40:[p(1)+p(3)+p(2)+1]/6

39:p(4)=>43=>44-49(equal)

p(3)=>42

p(2)=>41

p(1)=>40

1/6 =>44-45

39:[p(1)+p(2)+1]/6

38:p(5)=>43=>44-49(equal)

p(4)=>42

p(3)=>41

p(2)=>40

p(1)=>39

1/6 =>44

38:[p(1)+1]/6

So 44 is the most probable number

（3）真的不会。。。TAT。。

p(1)=1/6

p(2)=1/6

p(3)=1/36+1/6=7/36

[21,3]

p(4)=1/216+2/36+1/6=49/216

[211,22,31,4]

p(5)=1/1296+3/216+2/36+1/6=307/1296

[2111,311,212,221,23,32,5]

p(6)=1/7776+4/1296+5/216+3/36+1/6=2149/7776

[21111,3111,51,6,2211(3),222,231(4),24,33]

To be continued

**Q5：**

1/9;5/18;11/18

Assume X, Y ~ U(0,1) i.i.d.

Let A=min[X,Y]

B=max[X,Y]

C=min(A,1-B,B-A)

Fc(a) denote cdf and fc(a) denote pdf

1−Fc(a)=P(C≥a)=P(A≥a,1−B≥a, B−A≥a)= P(a≤X≤1−a,a≤Y≤1−a,|X−Y|≥a)

=>{(x,y):0≤x,y≤1,y≤x−a,x≤1−a,y≥a} & {(x,y):0≤x,y≤1,y≥x+a,x≥a,y≤1−a}

To ensure these sets are non-empty, 0≤a≤1/3. The sum of two sets are (1-3a)^2

So 1−Fc(a)= (1-3a)^2 =>Fc(a)=1-(1-3a)^2 => fc(a)=6a-18a^2

Smallest: E(C)= da=1/9

Now Let C=max(A,1-B,B-A)

Fc(a)=P(C≤a)=P(A≤a,1−B≤a, B−A≤a)

Suppose X≤Y Then =>P(0≤X≤a, 1≥Y≥1-a,Y-X≤a)

If a≥1-a, then 1≥a≥1/2: [1-3(1-a)^2]\*0.5

If a≤1-a, then 1/2≥a≥1/3:0.5\*(3a-1)^2

If Y≤X, then the same.

So when 1≥a≥1/2: Fc(a)= 1-3\*(1-a)^2, fc(a)=6-6a

When 1/2≥a≥1/3: Fc(a)=(1-3a)^2, fc(a)=18a-6

So Biggest: = 11/18

So Middle=1-1/9-11/18=5/18